Vectors and Coordinates

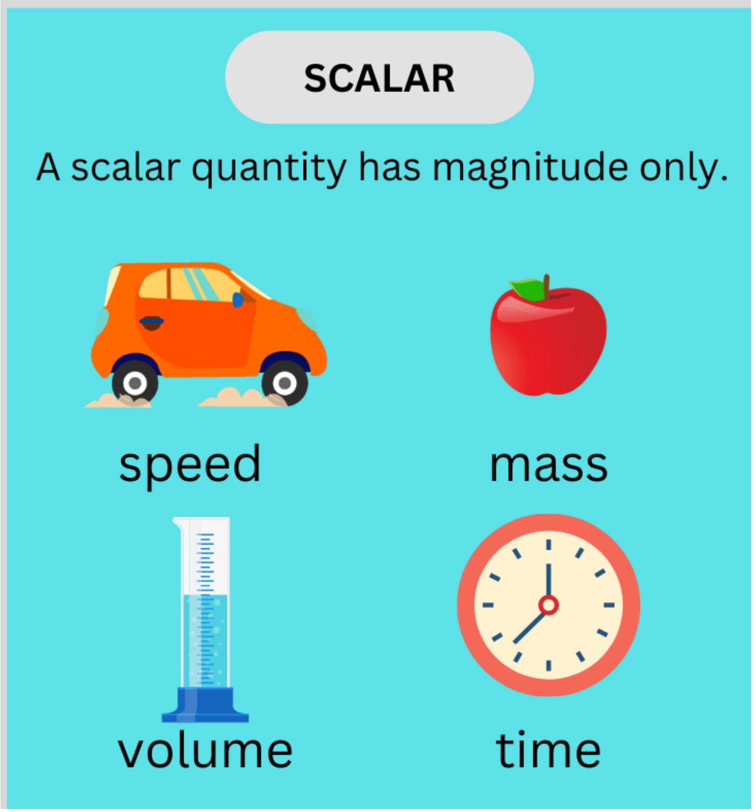
YORK COLLEGE & UNIVERSITY CENTRE

Scalar quantities

- **Definition:** Scalars are quantities that have magnitude only (size or amount), with no direction.

- **Examples:**
  - Mass (e.g. 5 kg)
  - Temperature (e.g. 22 °C)
  - Time (e.g. 3 s)
  - Speed (e.g. 10 m/s)
  - Energy (e.g. 200 J)

- **Key Point:** Scalars can be added or subtracted using normal arithmetic.



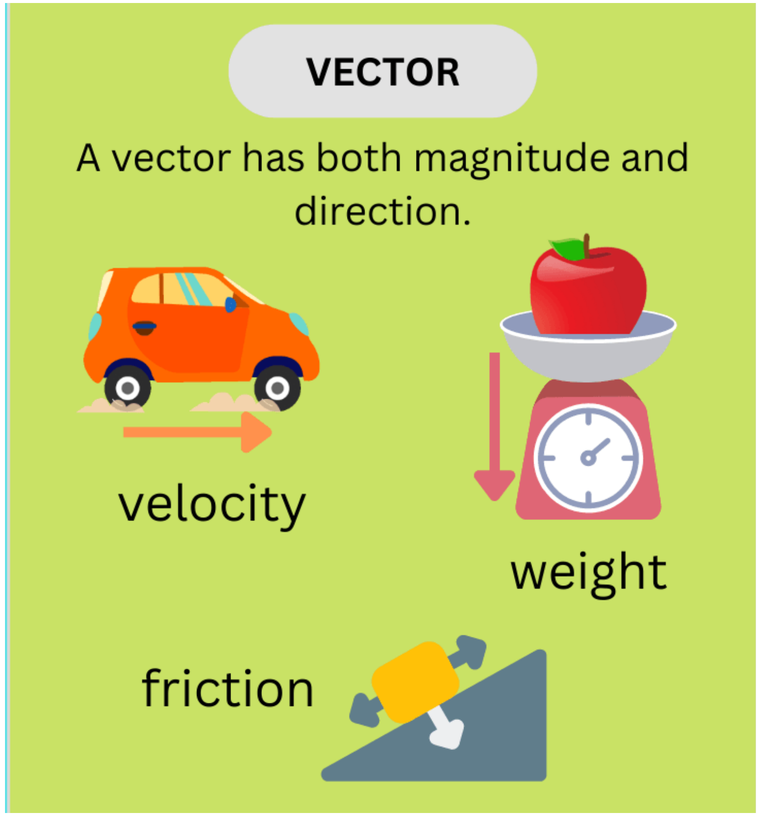
Vector quantities

- **Definition:** Vectors are quantities that have magnitude and direction.

- **Examples:**
  - Displacement (e.g. 5 m East)
  - Velocity (e.g. 20 m/s North)
  - Force (e.g. 10 N downward)
  - Acceleration (e.g. 3 m/s^2 to the left)

- **Key Point:** Vectors must be added using vector rules

- **Notation:** Often shown with arrows (→) or bold letters



Scalar and vector quantities

Scalar Quantities (magnitude only)

Mass (kg)

Temperature (°C, K)

Time (s)

Speed (m/s)

Energy (J)

Distance (m)

Volume (m^3)

Density (kg/m^3)

Power (W)

Pressure (Pa)

Vector Quantities (magnitude + direction)

Displacement (m, with direction)

Velocity (m/s, with direction)

Force (N, with direction)

Acceleration (m/s^2, with direction)

Momentum (kg·m/s, with direction)

Weight (N, acts toward Earth's centre)

Torque (N·m, with rotation direction)

Electric Field (N/C, with direction)

Magnetic Field (Tesla, with direction)

Angular Velocity (rad/s, with rotation axis)

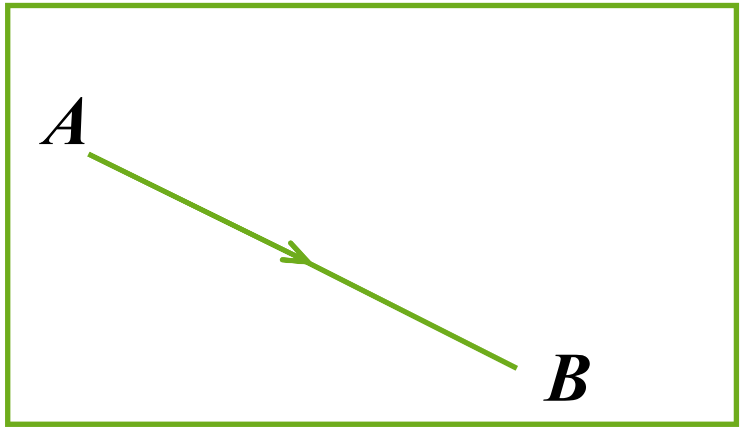
Drawing Vectors and Notation

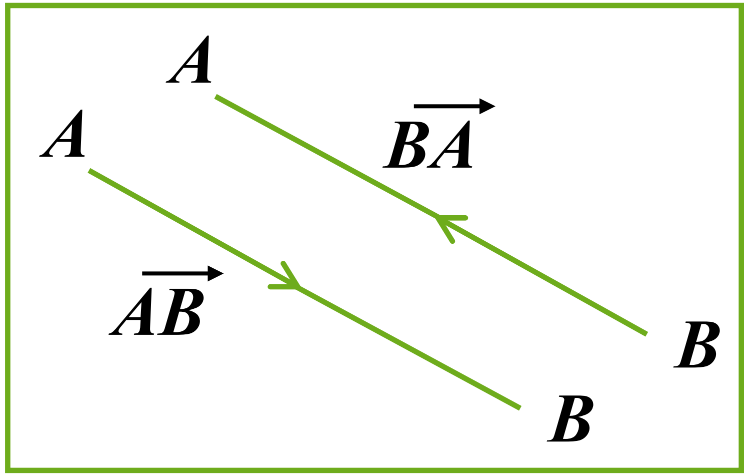
- Vectors can be drawn as a segment between two points with an arrow

- They can then be written as $\overrightarrow{AB}$

- Arrows that run in opposite directions can be written as inverses:

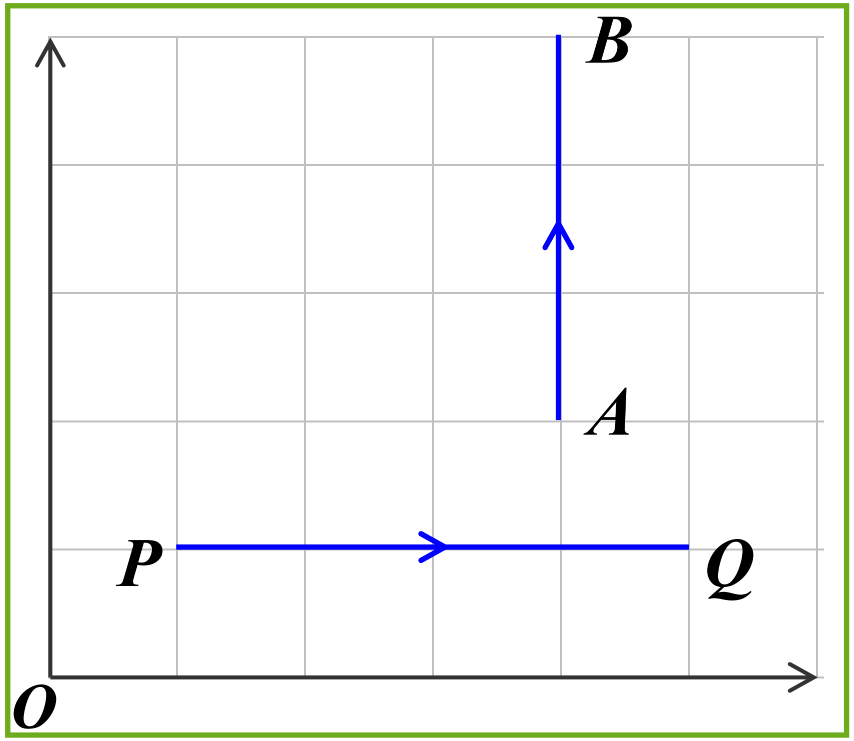
- $\overrightarrow{AB} = -\overrightarrow{BA}$





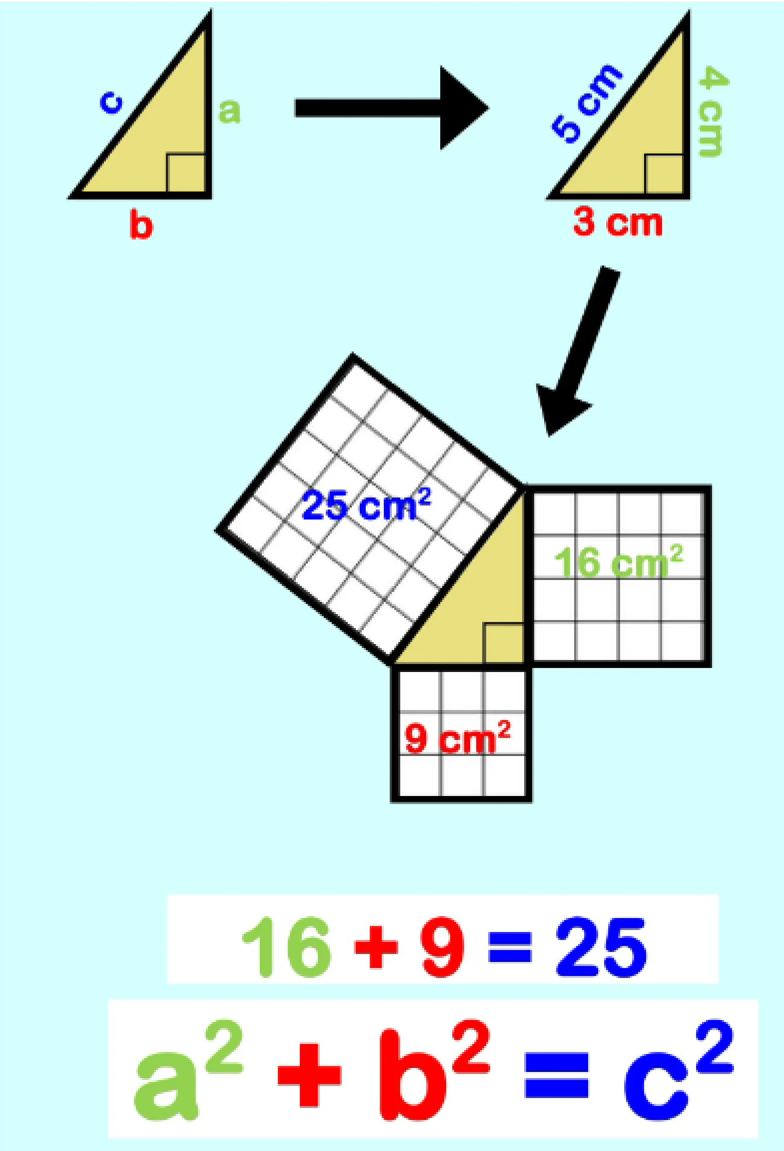
Magnitude of a vector

- The magnitude of a vector is based on the length of the line
- The grid on this slide has 1cm squares
- $\overrightarrow{AB}$ has the magnitude 3 so we can write AB = 3



Pythagorean Theorem Recap

- We know $a^2 + b^2 = c^2$
- This can be applied to vectors too
- Vectors are essentially just hypotenuses of triangles
- So to work out the magnitude of a vector we can use Pythagorean theorem



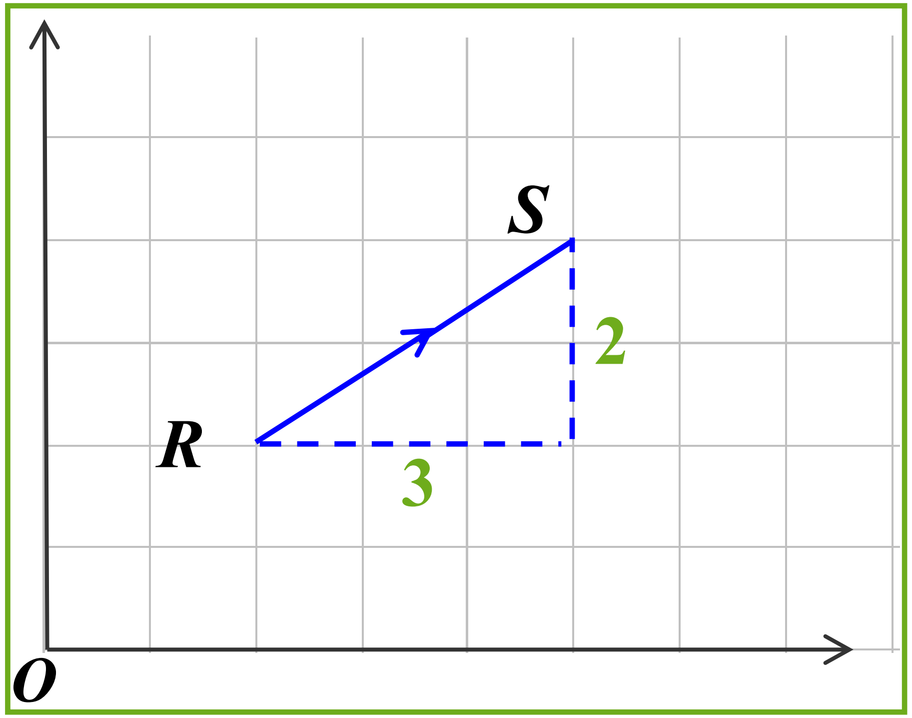
Calculating the magnitude of vectors

- Using Pythagoras' theorem we can take x and y and work out the magnitude

- $RS^2 = 3^2 + 2^2$

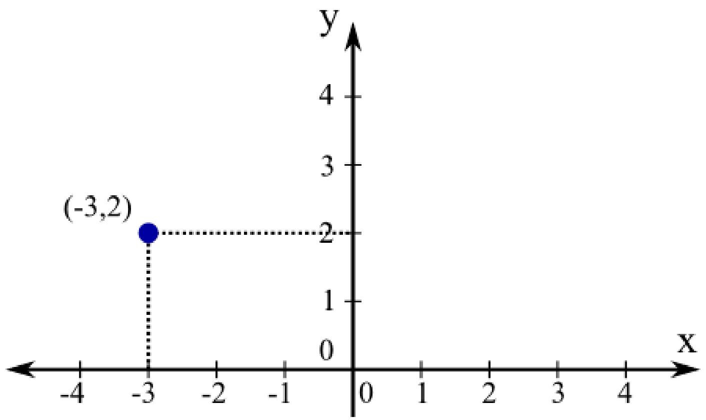
- $RS^2 = 13$

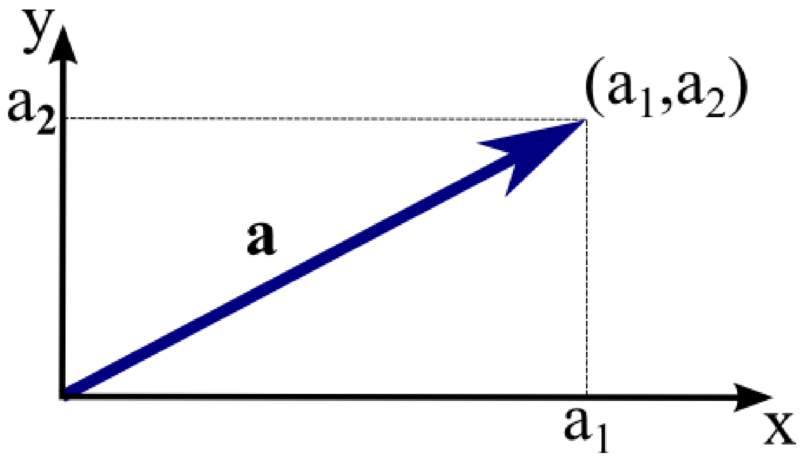
- $RS \approx 3.61$



Cartesian Coordinates & Vectors

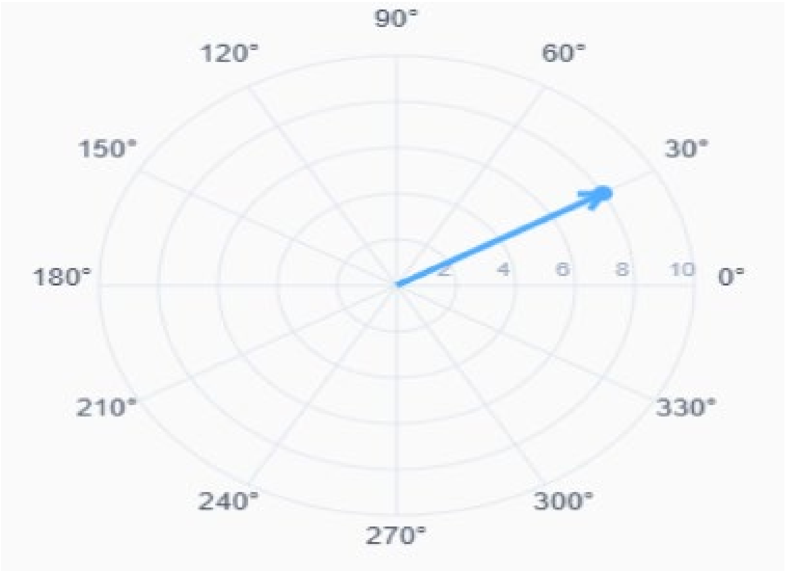
- **Cartesian Coordinates:**
  - Use x, y, (and z) axes to describe positions in 2D or 3D space.
  - A point is written as an ordered set:
    - 2D → (x, y)
    - 3D → (x, y, z)
- **Vectors in Cartesian Form:**
  - Expressed in terms of components along each axis.
  - Example (2D): v = (3, 4) → 3 units in x, 4 units in y.
  - Example (3D): F = (2, –1, 5) → 2 in x, –1 in y, 5 in z.





Polar Coordinates & Vectors

- Polar Coordinates:
  - Describe a point using radius (r) and angle (θ) instead of x and y.
  - A point is written as: (r, θ).
    - Example: (5, 30°) → 5 units from origin, at 30° counter-clockwise from x-axis.
- Vectors in Polar Form:
  - Expressed by magnitude and direction.
  - Example: displacement = 10 m at 45°.
  - It can also be written as magnitude ∠ angle $(10\angle 45^o)$



Cartesian to polar

$$\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(x) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(x) = \frac{\text{opposite}}{\text{adjacent}}$$

- To find the magnitude we use the calculation we looked at earlier
- To find the angle we just put use $S_h^o C_h^a T_a^o$ we know the adjacent and opposite values as they are x and y
- So, we can use tan to find the angle
- Make sure you are using the right option on your calculator too, either radians or degrees depending on the question

Convert Cartesian to Polar

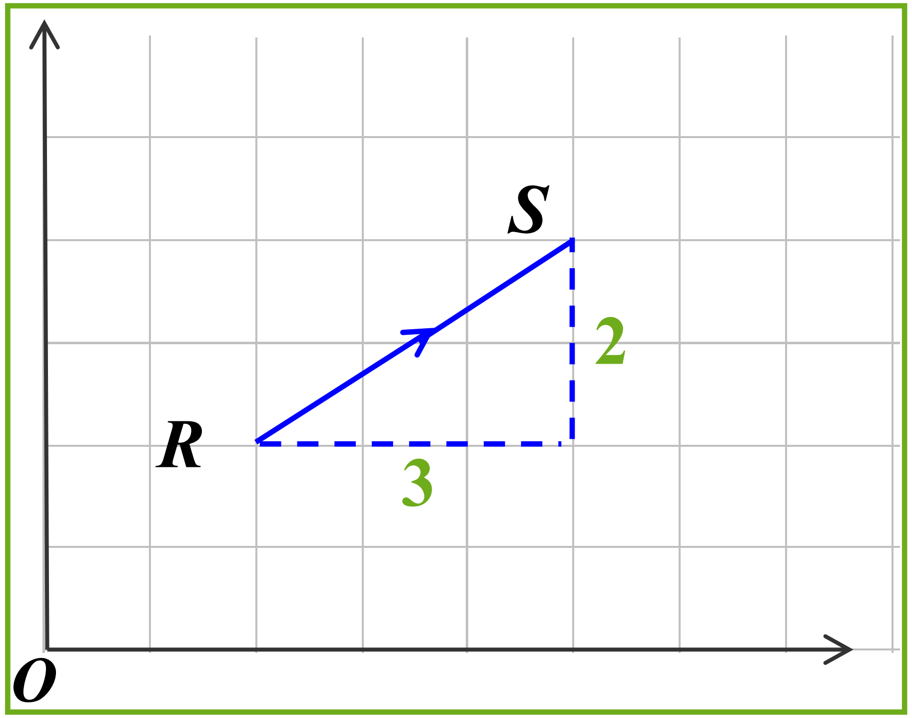
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

note: may need to add 180°

Cartesian to polar

- So, we know that $RS^2 = 3^2 + 2^2$
- $RS \approx 3.61$
- $\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.69006753^o$
- So our final polar form is:
- $3.61\angle 33.69^o$



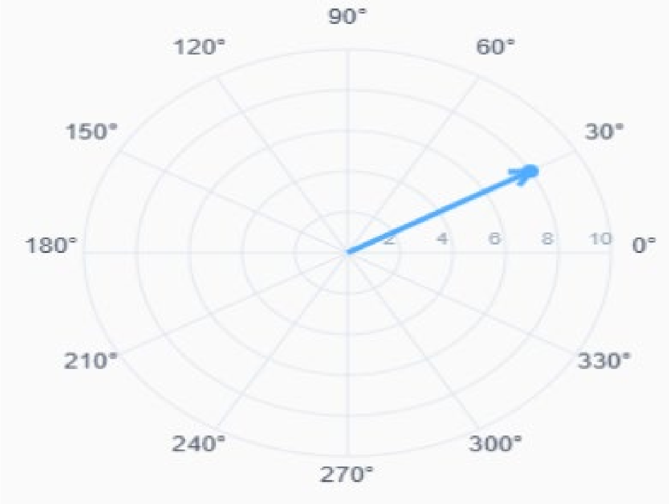
Polar to Cartesian

- To convert polar to cartesian again we just use $S_h^o C_h^a T_a^o$

- As we have the hypotenuse and the angle we can just cos and sin to work out x and y

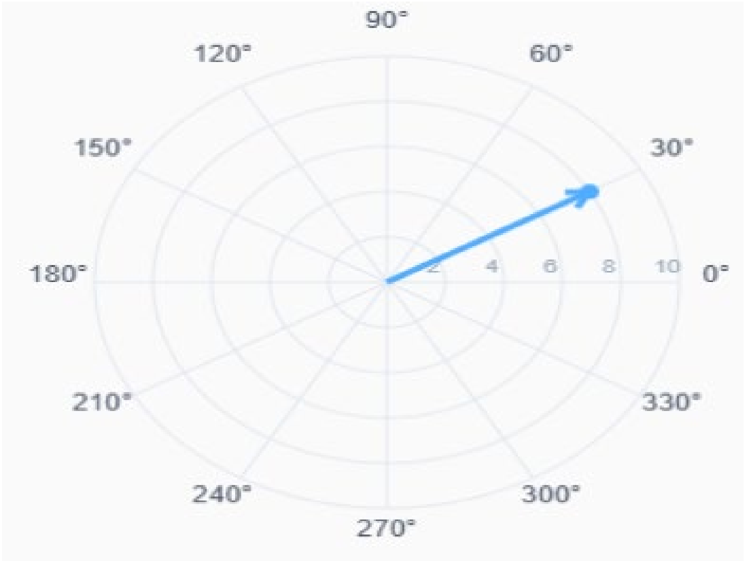
- $h * \sin\theta = y$

- $h * \cos\theta = x$



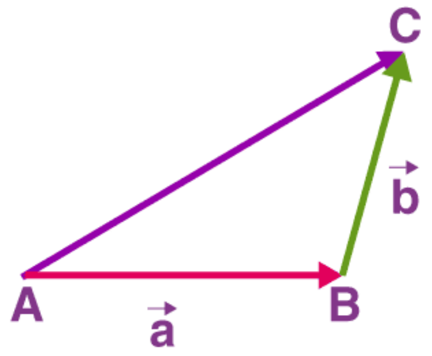
Polar to Cartesian

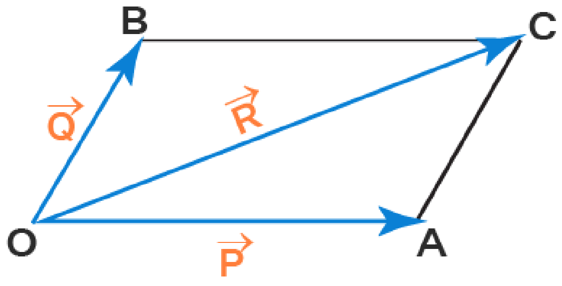
- So, for the graph on this slide we have:
- $8\angle 30^o$
- $x = 8 * \cos(30) = 6.92820323$
- $y = 8 * \sin(30) = 4$
- So, we can write: $(6.928, 4)$



Adding Vectors

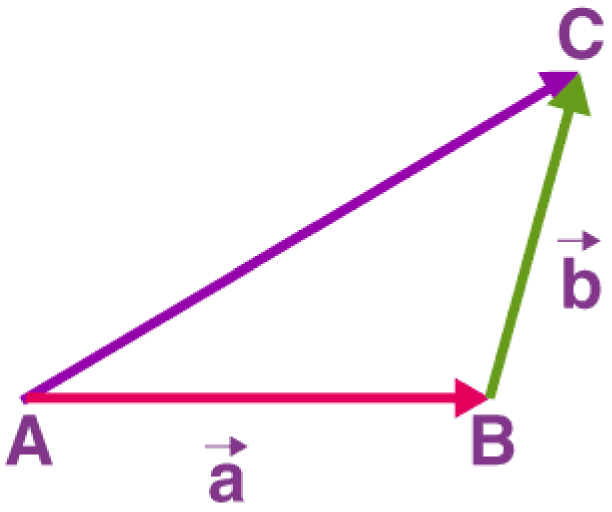
- There are several methods to add two vectors together
- There are two main graphical methods:
  - Tip-to-Tail Method
  - Parallelogram Method
- And one main mathematic method:
  - Component addition
- **Note graphical methods only work when drawing to scale**





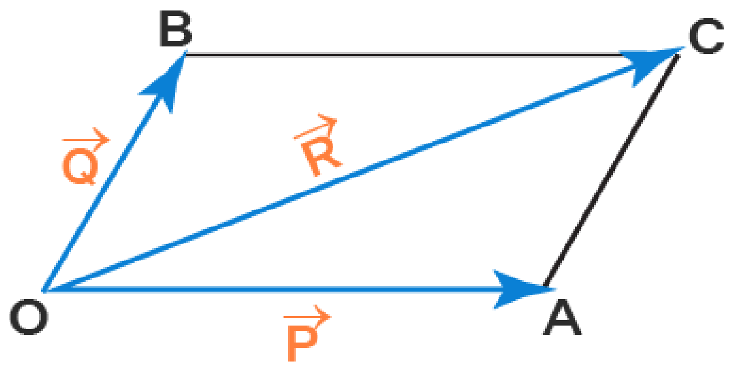
Tip to Tail Method

- In tip to tail method you draw one vector on a graph
- Then draw the next vector extending from the tail of the original
- We can repeat this for however many vectors you want to add
- Then we can measure the final line and work out the angle using a ruler and a protractor



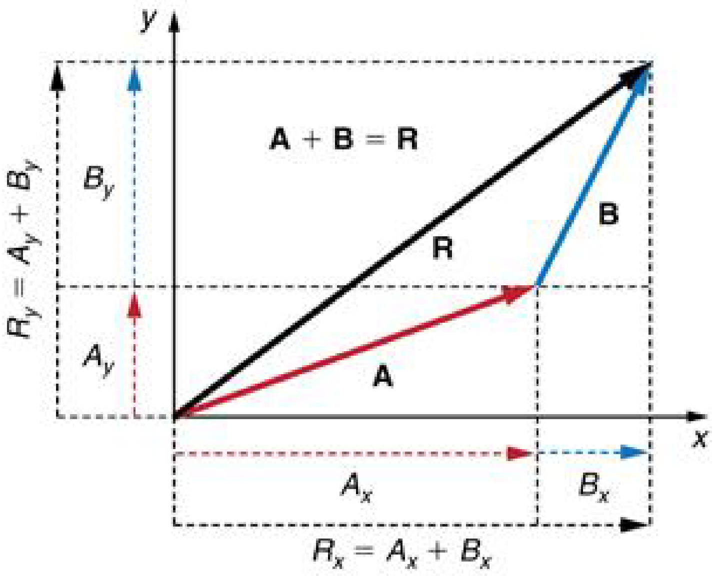
Parallelogram Method

- In the parallelogram method we draw out the vectors we are adding starting at the 0 point
- We then take these two lines to draw a parallelogram
- We then work out the distance from the 0 point to the far end of that parallelogram
- We can also work out the angle for this line for the resultant vector



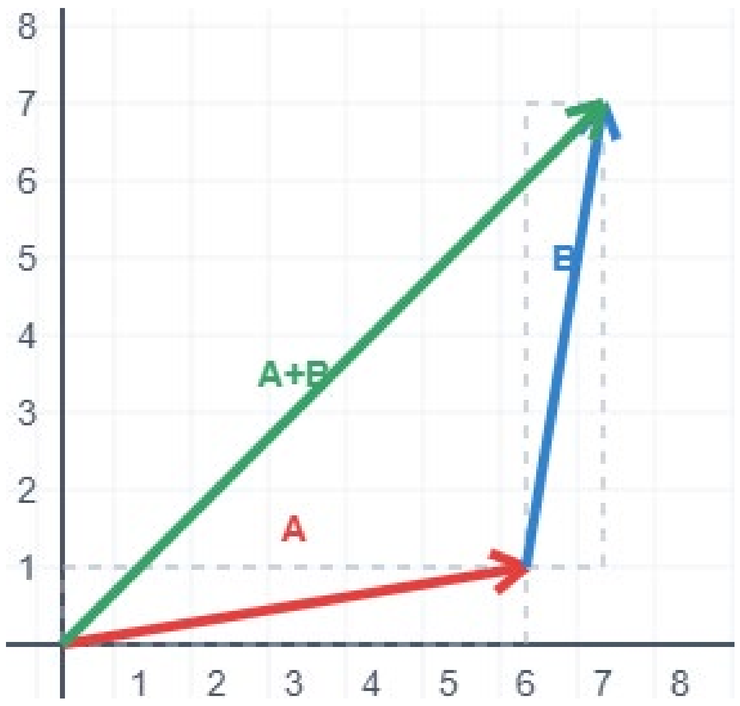
Component Addition

- For component addition we split both vectors into X and Y
- We then add both Xs and both Ys to give us a final resultant cartesian vector
- We can then do cartesian to polar conversion to find a resultant vector



Component addition example

- We can look at the graph on this page, we know that A is (6,1)
- We can also determine that B is (1,6)
- So, if we add our x components: 6+1 = 7
- Then if we add our y components: 1+6 = 7
- Then we get the final vector (7,7)



i and j

- If we don't want to constantly draw out our vectors or clarify them with $\overrightarrow{AB}$ we can write out a vector with i and j
- i is any movement in the x axis
- j is any movement in the y axis
- So we can write $v = 3i + 2j$