AC Circuit Analysis

York College & University Centre

Circuit with Resistors

- We know resistors don't change the phase of the voltage or current.
- We can just use these equations to work out the values in a resistive circuit
  - Voltage = Current * Resistance
  - Power = Voltage * Current

$V = I * R$
$120 = I * 100$
$I = 1.2A$

$P = V * I$
$P = 120 * 1.2$
$P = 144W$

Circuits with Inductors

- For circuits with inductors, we have a phase shift

- This means we need to change our equations:

  - Inductive Reactance = 2π* Frequency * Reactance

  - Voltage = Current * Inductive Reactance

$$X_L = 2\pi f L$$
$$X_L = 2\pi * 1000 * 5$$
$$X_L = 31415.92654$$

$$V = I X_L$$
$$120 = I * 3141592654$$
$$I = 0.003819718634$$

Circuits with Capacitors

- For circuits with capacitors, we again have a phase shift

- This means we need to use the equations:
  - Capacitive Reactance = 1/2π*frequency*capacitance
  - Voltage = Current * Capacitive Reactance

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2\pi * 1000 * 5}$$

$$X_C = 3.18309886 * 10^{-5}$$

$$V = IX_C$$

$$120 = I * 3.18309886 * 10^{-5}$$

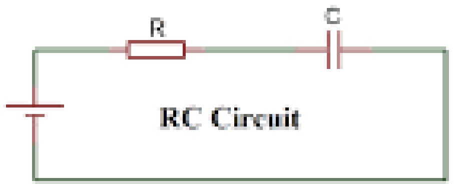
$$I = 3769911.184$$

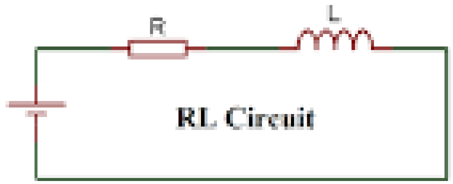
RLC, RC and RL Circuits

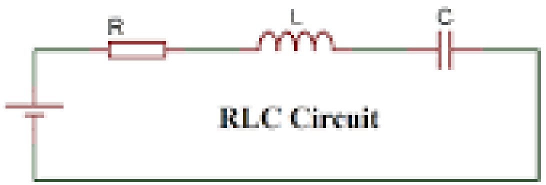
- RLC Circuits contain all 3 components;
  - A Resistor (R)
  - An Inductor (L)
  - A Capacitor (C)

- RL Circuits contain two of the components;
  - A Resistor (R)
  - An Inductor (L)

- RC Circuits contain two of the components;
  - A Resistor (R)
  - A Capacitor (C)







Series RL, RC and RLC Circuits

- For each of the different series circuits you can use impedance to work out voltage and current using the equation:

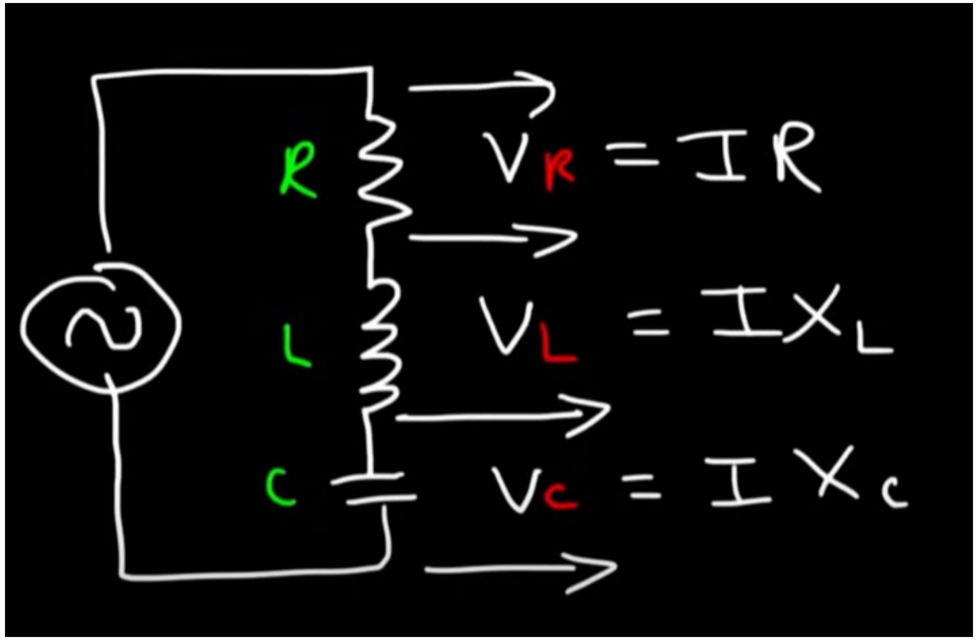
  - $Z = \sqrt{R^2 + (X_L - X_C)^2}$ where:

    - Z = Impedance
    - R = Resistance
    - X_L = Inductive Reactance
    - X_C = Capacitive Reactance

    - Note when you don't have an inductor or a capacitator you just set them to 0

Series RL, RC and RLC Circuits

- We can then work out Voltage or Current for the whole circuit using the equation
  - $V = IZ$
- However, for each component in the circuit we use their individual equation

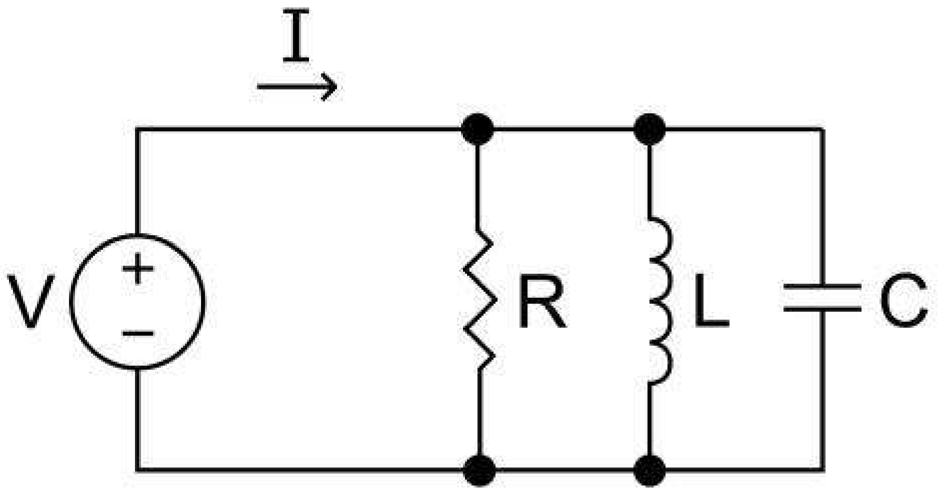


Parallel RLC Circuits

- When we have components in parallel, we work out our impedance differently

- We use the equation: $Z = \frac{1}{\sqrt{\frac{1}{R}^2 + (\frac{1}{X_L} - \frac{1}{X_C})^2}}$

- Remembering we can sub in 0 if we are missing a capacitor or an inductor

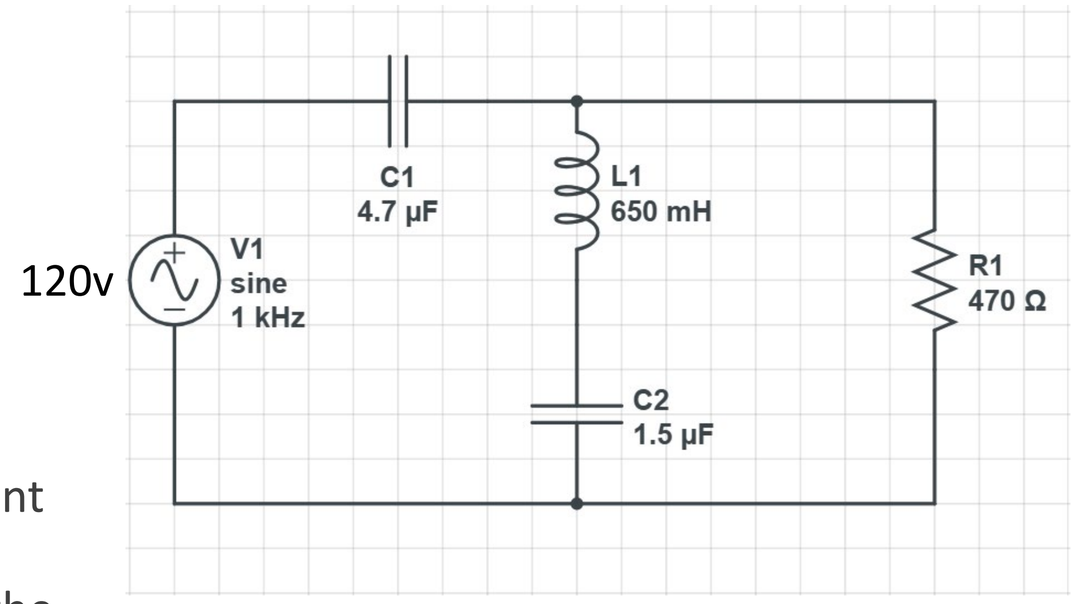


Example Question

- In this question we want to find the voltage and current through every component

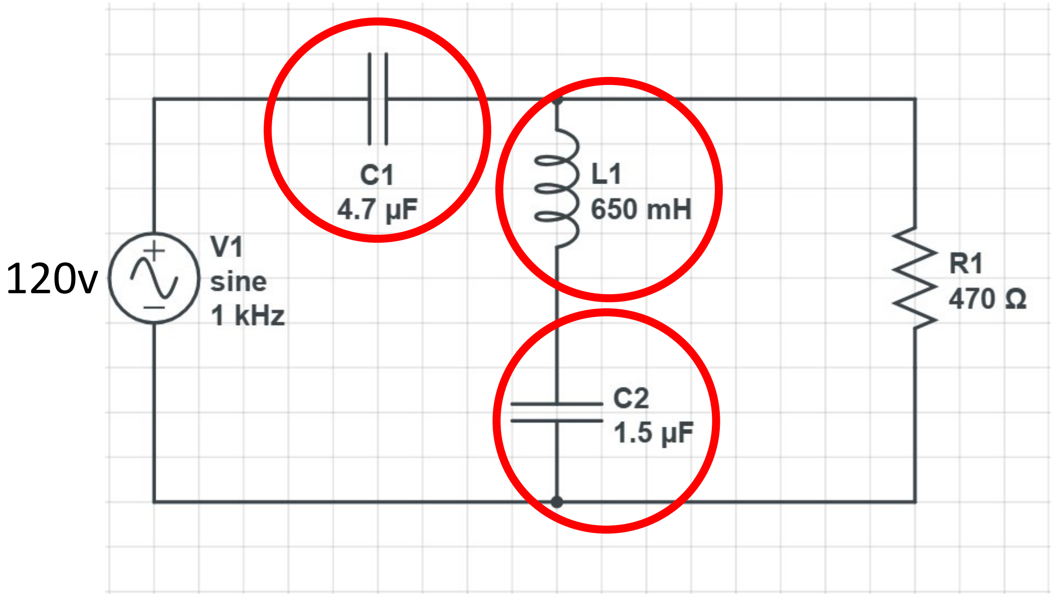
- The best strategy to take are to:

1. Combine components to make equivalent circuits
2. Continue combining components until the circuit is one equivalent component
3. Expand circuit back out calculating V and I for the components or groups as we go



Example Question

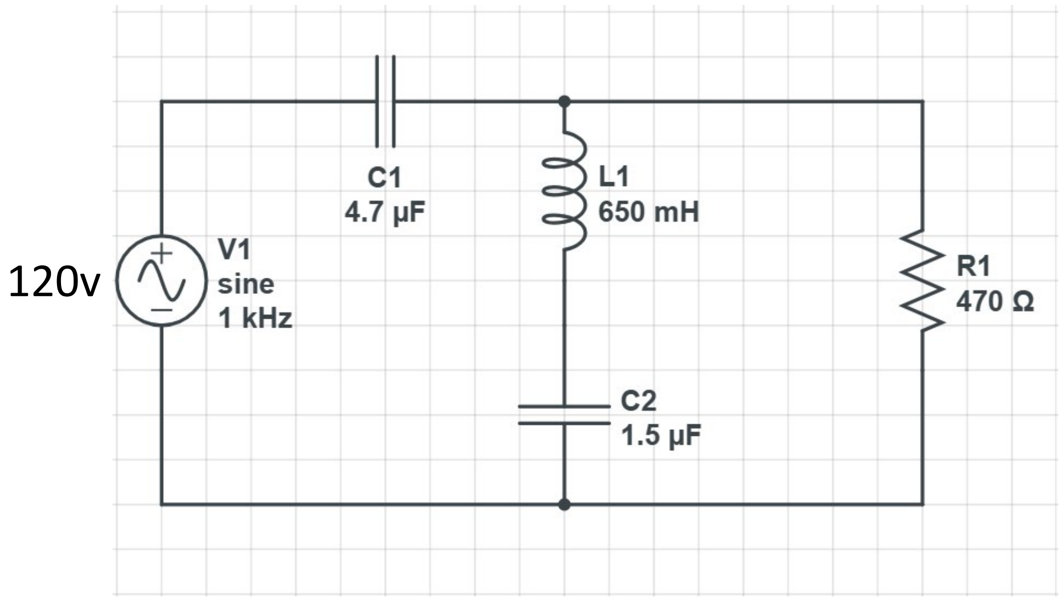
- Let's start by calculating the reactance for each component
- $X_{C1} = \frac{1}{2\pi fC} = \frac{1}{2\pi * 1000 * 0.0000047} = 33.86275385$
- $X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi * 1000 * 0.0000015} = 106.1032954$
- $X_{L1} = 2\pi fL = 2\pi * 1000 * 0.65 = 4084.07045$



Example Question

- Next let's write out our reactance's as impedances

  - $Z_{C1} = 33.863\angle -90^o$
  - $Z_{C2} = 106.103\angle -90^o$
  - $Z_{L1} = 4084.070\angle 90^o$
  - $Z_{R1} = 470\angle 0^o$



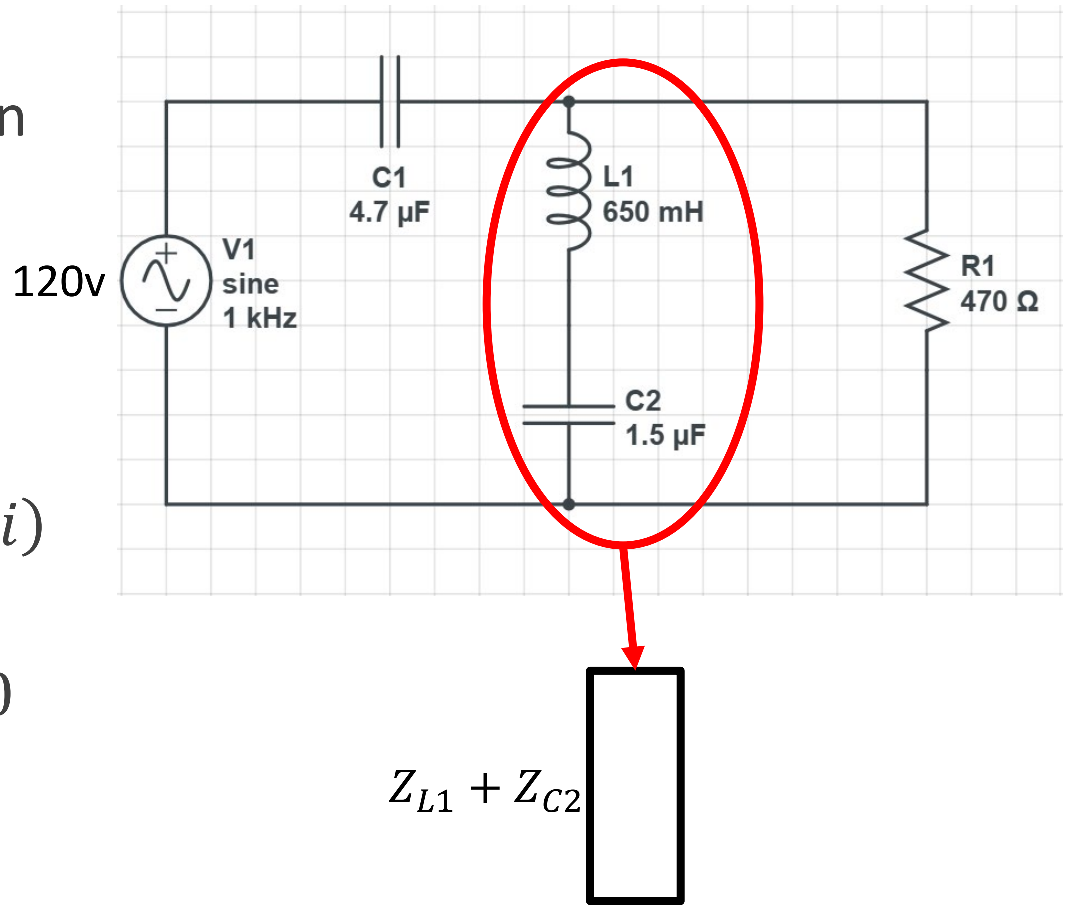
Example Question

- Next let's combine L1 and C2 into an equivalent component

- $Z_{L1} + Z_{C2}$

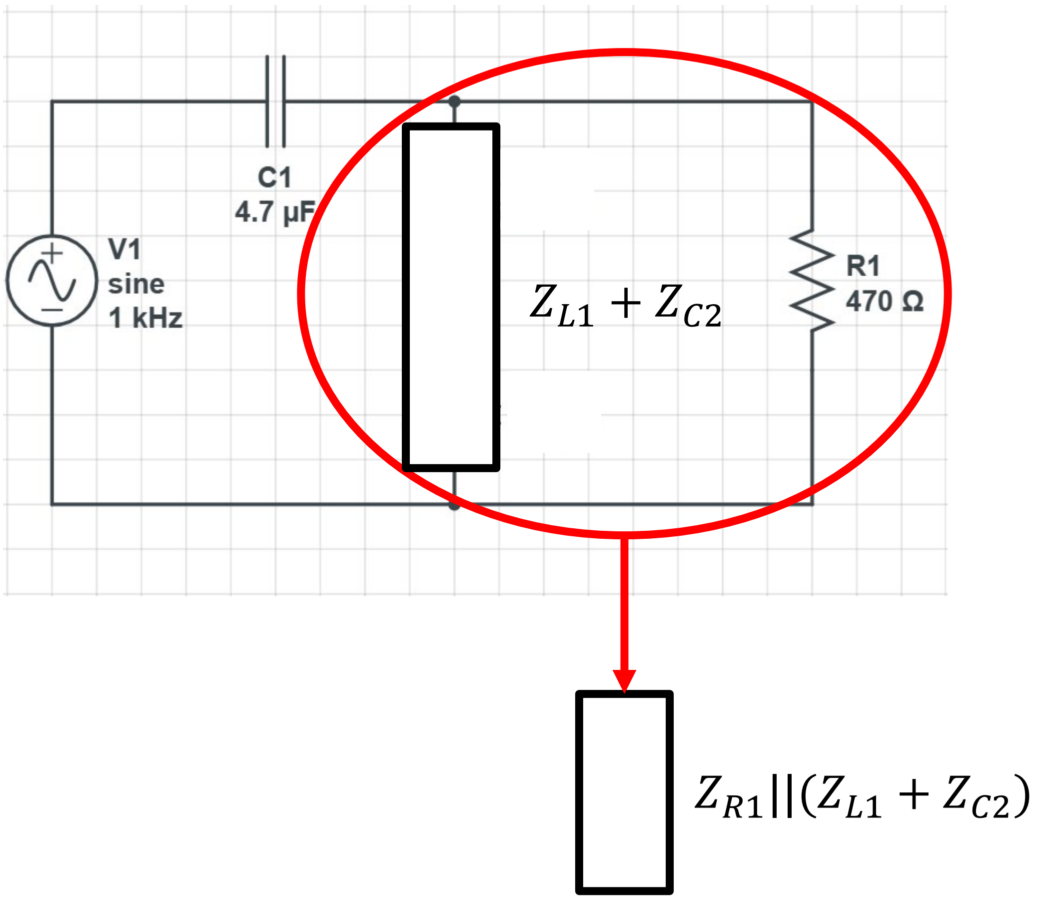
- $(0\ + 4084.070i) + (0\ - 106.103i)$

- $(0\ + 3{,}977.967i) = 3977.967\angle 90$



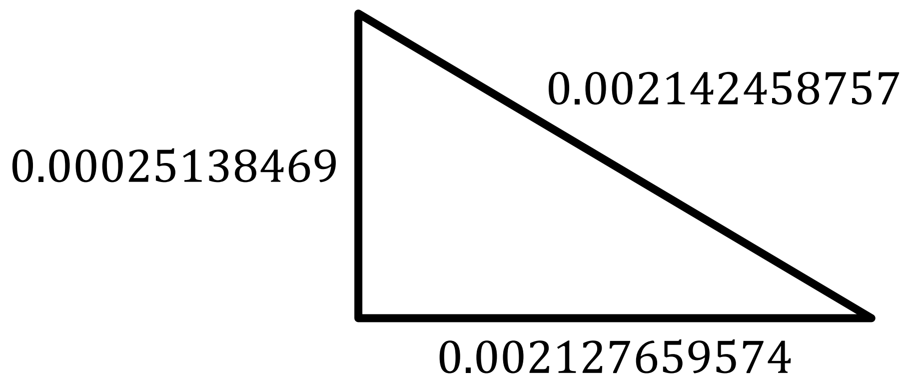
Example Question

- Next let's combine R1 with our equivalent component
- $Z_{R1}||(Z_{L1} + Z_{C2})$
- $\frac{1}{(\frac{1}{470\angle 0}+\frac{1}{3977.967\angle 90})}$
- $\frac{1}{(0.002127659574\angle 0+0.00025138469\angle 90)}$



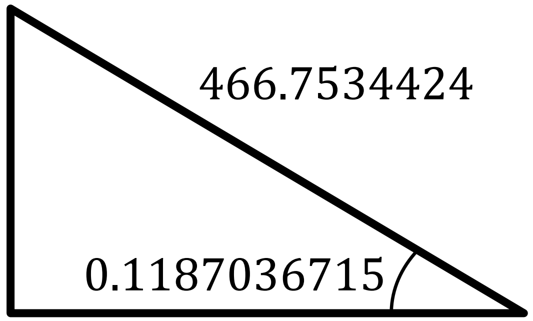
Example Question

- $\frac{1}{(0.002127659574\angle 0+0.00025138469\angle 90)}$
- $\sqrt{0.002127659574^2 + 0.00025138469^2}$
- $\frac{1}{0.002142458757} = 466.7534424$
- $\tan(\frac{0.00025138469}{0.002127659574}) = 0.1187036715$
- $466.7534424\angle 0.1187036715$



Example Question

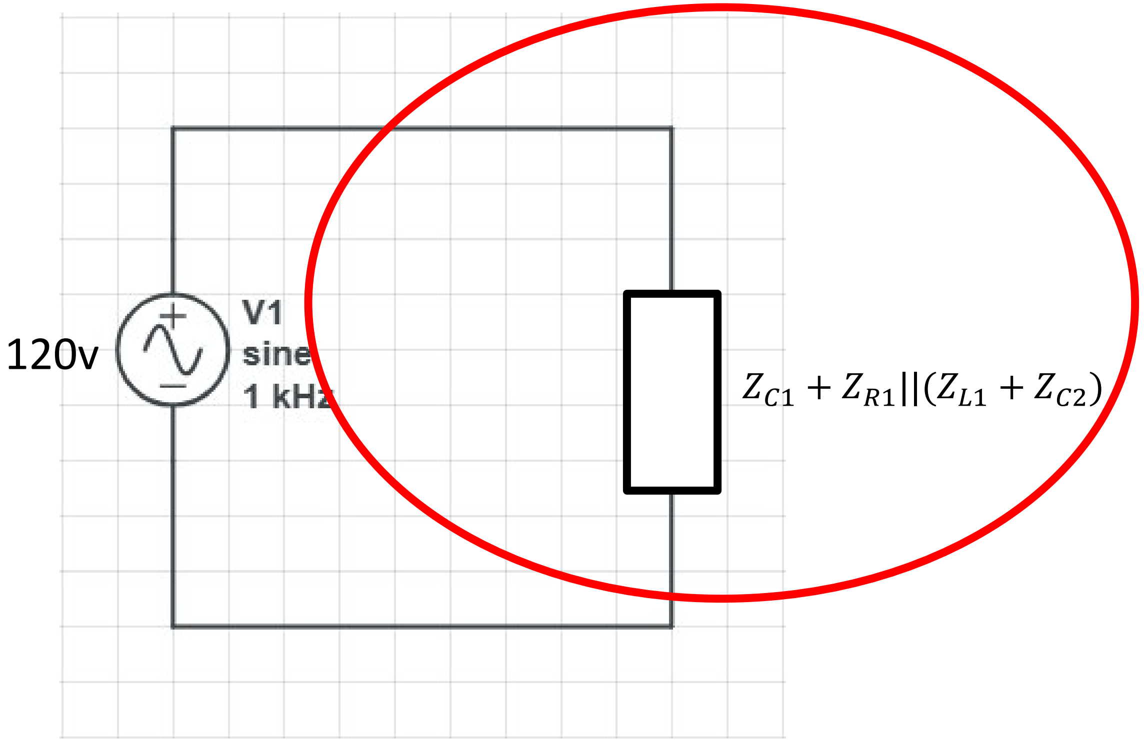
- $466.7534424\angle 0.1187036715$
- Let's convert this back to rectangular form
- $466.7534424 * \sin(0.1187036715) = 55.27532353i$
- $466.7534424 * \cos(0.1187036715) = 463.4688928$
- $(463.4688928 + 55.27532353i)$



$s_h^o c_h^a t_a^o$

Example Question

- Let's combine our final two components into an equivalent component
- $Z_T = Z_{C1} + Z_{R1}||(Z_{L1} + Z_{C2})$
- $(0 + 33.86275385i) + (463.4688928 + 55.27532353i)$
- $(463.4688928 + 89.13807738i)$
- $\sqrt{463.4688928^2 + 89.13807738^2} = 471.9629344$
- $\tan\left(\frac{89.13807738}{463.4688928}\right) = 0.1947351008$
- $471.9629344\angle 0.1947351008$



Example Question

- Let's work out the current going through this combined component
- $V = IZ$
- $I = \frac{V}{Z}$
- $I = \frac{120\angle 0}{471.9629344\angle 0.1947351008} = 0.25425725465$