Polynomials

YORK COLLEGE & UNIVERSITY CENTRE

What is a polynomial?

- A mathematical expression made up of variables (also called indeterminates) and constants

- Combined using only addition, subtraction, and multiplication

- Some of the variables have nonnegative integer exponents

Polynomial formatting

- They follow the format:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

- Where:
  - The “a” values are coefficients ($a_n \rightarrow a_0$)
  - X is the variable
  - n is a nonnegative integer

Examples of Polynomials

- Some examples of Polynomials are:
- $3x^2 + 2x - 1$ (this is a quadratic as its highest exponent is 2)
- $10x^3 + 2x^2 + x - 2$ (this is a cubic polynomial)
- $3x^4 + 2x + 2$ (this is a quartic polynomial)
- The last one is missing x^3 and x^2 but we can think of those as having a coefficient of 0 so it's the same as:
- $3x^4 + 0x^3 + 0x^2 + 2x + 2$

Adding & Subtracting Polynomials

- Adding polynomials is super easy, all you do is add together “like terms”
- You do the same for subtraction just taking away rather than adding
- Its easier when subtracting to put the negative into the polynomial then add

$$
\begin{array}{r}
3x^2 - 4x + 8 \\
+ \; -1x^2 - 2x + 3 \\
\hline
\boxed{2x^2 - 6x + 11}
\end{array}
$$

Combine like terms.

Example of adding polynomials

- $(2x^2+3x - 1)\ + (-3x^2 + 1x + 2)$

- I find it easier to write it in a table especially with bigger polynomials:

	$2x^2$	$3x$	-1
+	$-3x^2$	$1x$	2
	$-1x^2$	$4x$	1

- So, our final polynomial is $(-1x^2 + 4x + 1)$

Example of subtracting polynomials

- $(5x^2+2x - 10)\ - (-3x^2 + 8x + 2)$
- We need to then put the negative into the second polynomial:
- $-3x^2 \rightarrow +3x^2, \quad 8x \rightarrow -8x, \quad 2 \ \rightarrow -2$

	$5x^2$	$2x$	-10
+	$3x^2$	$-8x$	-2
	$8x^2$	-$6x$	-12

- So, our final polynomial is $(8x^2 - 6x - 12)$

Your Turn

Can you add these polynomials:

1. $(2x^2 + 3x + 2) + (6x^2 + 5x + 4)$
2. $(6x^2 - 3x - 2) + (-3x^2 + 4)$
3. $(x^3 + 3x^2 + 2) + (4x^2 - 5x - 4)$

Can you subtract these polynomials:

4. $(10x^2 + 6x + 16) - (2x^2 + 5x + 4)$
5. $(9x^2 - 5x - 8) - (3x^2 - 2)$
6. $(7x^3 + 4x^2 + 2) - (2x^2 + 5x - 4)$

Single Value Multipliers

- When we have a value outside of the bracket, we just multiply it by each term

- So, if we have $2(4x^2 + 2x + 1)$

- We get $\left((2 * 4x^2) + (2 * 2x) + (2 * 1)\right)$

- Which is the same as: $(8x^2 + 4x + 2)$

Multiplying Polynomials

- If we wish to multiply multiple binomials together, we multiply each term in one polynomial by each other term

- $(ax^2 + bx + c) * (dx^2 + ex + f) =$

$$ax^2(dx^2 + ex + f) + bx(dx^2 + ex + f) + c(dx^2 + ex + f)$$

- Then we use the single value multiplication method and then combine like terms

Example of Multiplying Polynomials

- $(4x^2 + 2x + 7) * (3x^2 + 7x + 2) =$

$$4x^2(3x^2 + 7x + 2) + 2x(3x^2 + 7x + 2) + 7(3x^2 + 7x + 2)$$

- $4x^2(3x^2 + 7x + 2) = 12x^4 + 28x^3 + 8x^2$
- $2x(3x^2 + 7x + 2) = 6x^3 + 14x^2 + 4x$
- $7(3x^2 + 7x + 2) = 21x^2 + 49x + 14$
- Then we just combine like terms.

Example of Multiplying Polynomials

- $12x^4 + 28x^3 + 8x^2 + 6x^3 + 14x^2 + 4x + 21x^2 + 49x + 14$

- $x^4 \rightarrow 12$
- $x^3 \rightarrow 28 + 6 = 34$
- $x^2 \rightarrow 8 + 14 + 21 = 43$
- $x \rightarrow 4 + 49 = 53$
- +14

- So, we can write it as: $(12x^4 + 34x^3 + 43x^2 + 53x + 14)$

Exponentials on Polynomials

- When we have a polynomial to a power all we do is treat it like a multiplication
- $(4x^2 + 3x + 2)^2 = (4x^2 + 3x + 2) * (4x^2 + 3x + 2)$
- Then we just multiply the terms like we did before

Your turn

- Can you solve these questions:
- $(4x^2 + 10x + 4) * (8x^2 + 8x + 3)$
- $(2x^3 + 9x^2 + 5x + 8) * (3x^2 + 11x + 9)$
- $(9x^3 + 4x + 2)^2$

Dividing Polynomials

- To solve polynomial division there are 3 main methods:

- Factorisation
- Long division
- Synthetic division

- I'm going to use long division as I think its easiest but you can use whatever suits you.

Solving polynomial division

- If we have the question: $\frac{2x^2-x-6}{x-2}$

- First we put in our values like so: $x-2 \overline{\big)\ 2x^2-1x-6}$

- Then we divide the first variable by the divider: $\frac{2x^2}{x} = 2x$

$$\begin{array}{r|l} & 2x \\ \hline x-2 & 2x^2-1x-6 \end{array}$$

- Then we multiply the full divider by the result

$$\begin{array}{r|l} & 2x \\ \hline x-2 & 2x^2-1x-6 \\ & -(2x^2-4x) \end{array}$$

Solving polynomial division

- Next, we take away this new polynomial from the original
- So, now we continue our division ($\frac{3x}{x} = 3$)
- Then we again multiply our result by the full divider
- So our result is $2x + 3$

$$
\begin{array}{r|l}
 & 2x \\
\hline
x - 2 & 2x^2 - 1x - 6 \\
 & -(2x^2 - 4x) \\
 & (0 + 3x - 6)
\end{array}
$$

$$
\begin{array}{r|l}
 & 2x + 3 \\
\hline
x - 2 & 3x - 6
\end{array}
$$

$$
\begin{array}{r|l}
 & 2x + 3 \\
\hline
x - 2 & 3x - 6 \\
 & -(3x - 6)
\end{array}
$$

Engineering Polynomial Division Question

A mass-spring-damper system is subjected to an external force, and the equation of motion in the Laplace domain is given by:

$$X(s) = \frac{F(s)}{Ms^2 + Cs + K}$$

Given the following values:

- $M = 1$ kg (mass),
- $C = 5$ Ns/m (damping coefficient),
- $K = 6$ N/m (spring constant),
- The input force is $F(s) = s^3 + 4s^2 + 5s + 2$

a) Find the displacement $X(s)$ in terms of a rational function.

b) Use polynomial division to simplify $X(s)$ into a form that separates the polynomial part from the proper fraction part.