Integration

What is integration

- To solve an integration, you just do the inverse of what you do when finding the derivative
- This means integration follows the same rules that differentiation does but in reverse
- You will often hear it defined as the “antiderivative”

Integration representation

- When we solve an integration, we must use a set of symbols
- The first is the integral symbol $\int$ which we put before our expression to make it clear it's an integration
- The next is dx, dy, dt, etc. which we put after the expression to tell us the variable of integration
- So, a full integration might look like

Definite vs Indefinite Integration

- **Indefinite Integration:**
- Finds a general formula for the antiderivative.
- This means we use value c to represent any potential constant that we don't know
- We do this when we don't have limits
- For indefinite we use an empty integration symbol $\int$

- **Definite Integration:**
- Calculates the numerical area under a curve between two limits.
- This means we don't have to include value c as we are just finding a number not an expression
- We do this when we have limits from the question
- For definite we use a filled in integration symbol $\int_b^a$

Power rule (indefinite integration)

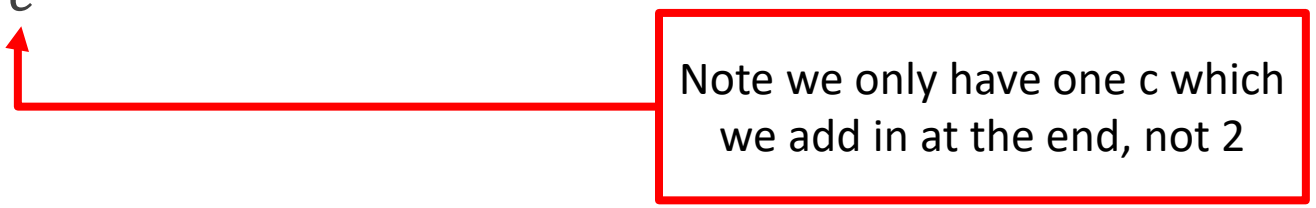
- The integration power rule is as such:
- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
- So, if we have x^4
- $\int x^4 \, dx = \frac{x^{4+1}}{4+1} + c = \frac{x^5}{5} + c$

Constant multiple rule (indefinite integration)

- The integration multiple rule is as such:
- $\int ax^n dx = a * \frac{x^{n+1}}{n+1} + c$
- So, if we have $20x^4$
- $\int x^4\, dx = \ 20 * \frac{x^{4+1}}{4+1} + c \ = \ 20 * \frac{x^5}{5} + c \ = \ 4x^5 + c$

Sum/difference rule (indefinite integration)

- The integration sum/difference rule is as such:
- $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$
- So, if we have $f(x) = 30x^2$ & $g(x) = 4x^3$ & $f(x) + g(x)$
- $\int [f(x) + g(x)] = 10x^3 + x^4 + c$



Integral of a constant(indefinite integration)

- The rule when integrating a constant is as such:
- $\int a\, dx = ax + c$
- So, if we have $f(x) = 5$
- $\int f(x) = 5x + c$

Exponential Rule (indefinite integration)

- The rule when integrating an exponential (e^x) is as such:
- $\int e^{ax}\, dx = \frac{1}{a} e^{ax} + c$
- So, if we have $f(x) = 5$
- $\int f(x) = 5x + c$

Trigonometric Integration (indefinite integration)

- The rules of trigonometric integration are:

- Note: Your calculator must be in radians mode for these equations as calculus always uses radians

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + C$$

$$\int \csc^2(ax)\,dx = -\frac{1}{a}\cot(ax) + C$$

$$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + C$$

$$\int \csc(ax)\cot(ax)\,dx = -\frac{1}{a}\csc(ax) + C$$

Definitive integration

- The primary rule in definitive integration is
- $\int_a^b f(x)dx = F(b) - F(a)$
- Where $F(x)$ is $\int f(x)dx$ (of course without the constant)

Example of Definitive integration

- $\int_0^5 15x^2 dx = F(5) - F(0)$
- $\int 15\,x^2 = 5x^3$
- $[5x^3]_0^5 = 5(5^3) - 5(0^3) = 625 - 0 = 625$