Exponentials

Multiplying Exponentials

- We know $x^2 = (x * x)$ and we know $x^4 = (x * x * x * x)$

- So if we do $x^2 * x^4$ we get $(x * x) * (x * x * x * x)$ which is $(x * x * x * x * x * x)$ or x^6

- So $x^n * x^m = x^{n+m}$

- Note this only works when the base is the same

$\Rightarrow$ With the Same Base

$a^m \times a^n = a^{m+n}$

Multiplying Exponentials

- When we have 2 values with he same exponential but different base we can put them in brackets
- If the bases and the powers are different then you can only multiply them

$\Rightarrow$ With Different Bases and Same Powers

$$a^n \times b^n = (a \times b)^n$$

$\Rightarrow$ With Different Bases and Different Powers

$$a^n \times b^m = (a^n) \times (b^m)$$

Using Logarithms

- When we have a value, we don't know as an exponential we can use logarithms

- This allows us to bring the exponential down and use it in our calculations

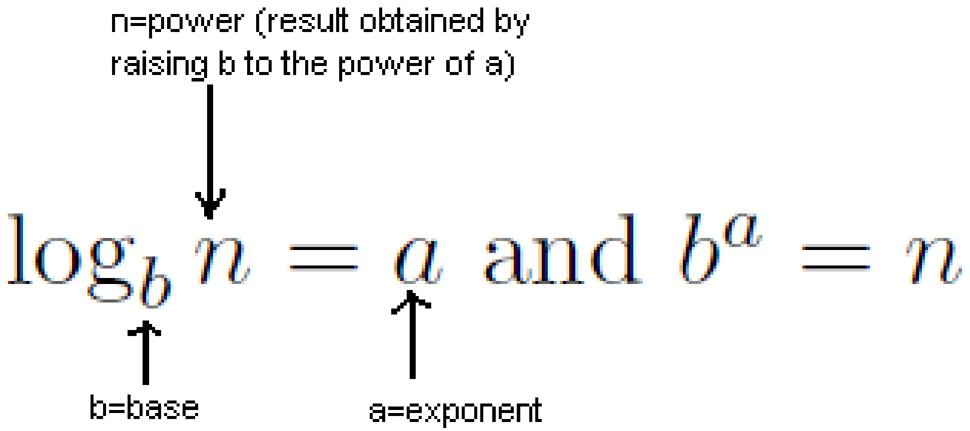
- Note if we are using logarithms, we must do the same to every part of our equation

$$y = \log_b(x)$$

$$b^y = x$$

Logarithm Bases

- A logarithm base is the value at the bottom of a logarithm
- By default, it is 10
- But when we solve exponentials, we substitute values into the bottom of the logarithm
- **Note:**
- $\log_x(x) = 1$
- So $\log_7(7) = 1$



Solving simple exponentials

- Our first step in solving a logarithm question is to log both sides using a log with base equal to the base of the exponential

- So, for 5^x we use $\log_5 5$ as it has the same base as the exponential

- We can then rearrange the equation to get our x value

1. $5^x = 20$
2. $x\log_5(5) = \log_5(20)$
3. $x = \frac{\log_5(20)}{\log_5(5)}$
4. $x = 1.861353116$

Solving simple exponentials

- This uses the same method we just employed; it's just a bit longer

- We start by logging both sides using $\log_{52}$ as the exponential has a base of 52 ($52^{(2x+3)}$)

- We then divide both sides by $\log_{52}(52)$ to isolate x

- We then isolate x further to get our final value

1. $52^{(2x+3)} = 29$

2. $(2x+3)\log_{52}(52) = \log_{52}(29)$

3. $2x + 3 = \dfrac{\log_{52}(29)}{\log_{52}(52)}$

4. $2x = \dfrac{\log_{52}(29)}{\log_{52}(52)} - 3$

5. $x = \dfrac{\frac{\log_{52}(29)}{\log_{52}(52)} - 3}{2} = -1.073894188$

Solving Simple Exponents (Part 2)

- This question is a bit harder, but we still use the same methodology
- We start by logging both sides, we can pick whatever base we want (either 10 or 15) but I always recommend whatever is easier
- In this situation 10 is easier so we use $\log_{10}$
- We then continue to simplify and rearrange the equation to get x

1. $10^{(4x+2)} = 15^{(3x+4)}$
2. $(4x+2)\log_{10}(10) = (3x+4)\log_{10}(15)$
3. $(4x+2) * 1 = (3x+4)\log_{10}(15)$
4. $4x + 2 = 3\log_{10}(15)x + 4\log_{10}(15)$
5. $4x - 3\log_{10}(15)x = 4\log_{10}(15) - 2$
6. $\left(4 - 3\log_{10}(15)\right)x = 4\log_{10}(15) - 2$
7. $0.4717262228x = 2.704365036$
8. $x = \frac{2.704365036}{0.4717262228} = 5.732912239$

A few questions

Find the value of X in all these questions:

1. $80^{(2x-20)} = 6\,400$
2. $32^{(2x+2)} = 10^{(3x-10)}$
3. $100^{(4x+9)} = 1\,000\,000^{x}$

1. $x = 11$

2. $x = -1\,263.141$

3. $x = -9$

Quick Solving Exponents

$$25^{(x+3)} = 625^{(2x+1)}$$

$$5^{2^{(x+3)}} = 5^{4^{(2x+1)}}$$

$$5^{2x+6} = 5^{8x+4}$$

$$2x + 6 = 8x + 4$$

$$6x = 2$$

$$x = \frac{2}{6} = \frac{1}{3} = 0.33333\ldots$$

e

- e is a mathematical constant

- e is an irrational number and thus goes on infinitely like pi

- e is approximately equal to **2.718281828**

$$e^{i\pi}+1=0$$

Euler's formula (not necessary but pretty)

Why is e important?

$$\log_e e^x = x$$

$$\log_e x = \ln x$$

This is known as the natural log

Using natural logarithms

$$e^{(2x+1)} = 10$$

$$2x + 1 = \ln 10$$

$$2x = \ln 10 - 1$$

$$x = \frac{\ln 10 - 1}{2} = 0.6512925465$$

Inverse Log

$$\log(100) = 2$$

$$\log^{-1}(2) = 100$$

Some questions

1. $10e^{(2x-2)} + 14 = 125$

2. $23^{5x} - 15 = 245$

3. $\log x = 82$